Fifth Semester B.E. Degree Examination, Jan./Feb. 2021 Information Theory and Coding

Time: 3 hrs .
Max. Marks: 100
Note: Answer any FIVE full questions, choosing ONE full question from each module.

## Module-1

1 a. Derive the expression for average information contents of symbols in long independent sequence.
(06 Marks)
b. Find the relationship between Hartley's, nats and bits,
(06 Marks)
c. A code is composed of dots and dashes. Assuming that a dash is 3 times as long as a dot and has one-third the probability of occurrence. Calculate:
(i) The information in a dot and dash
(ii) The entropy of dot-dash code
(iii) The entropy rate of information, if a dot losts for 10 ms and this time is allowed between symbols.
(08 Marks)

## OR

2 a. Consider a second order mark-off source as shown in Fig.Q2(a). Here $\mathrm{s}=\{0,1\}$ and states are $\mathrm{A}\{0,0\}, \mathrm{B}=\{0,1\}, \mathrm{C}=\{1,0\}$ and $\mathrm{D}=\{1,1\}$.
(i) Compute the probability of states
(ii) Compute the entropy of the source


Fig.Q2(a)
(10 Marks)
b. Prove that entropy of zero memory extension source is given by $\mathrm{H}\left(\mathrm{s}^{\mathrm{n}}\right)=\mathrm{nH}(\mathrm{s})$. ( $\mathbf{1 0}$ Marks)

## Module-2

3 a. A Discrete Memory Source (DMS) has an alphabet $X=\left\{x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}\right\}$ and source statistics. $\mathrm{P}=\{0.3,0.25,0.20,0.12,0.08,0.05\}$. Construct binary Huffman code. Also find the efficiency and redundancy of coding.
( $\mathbf{1 0}$ Marks)
b. Apply Shannon encoding algorithm to the following set of messages and obtain code
efficiency and redundancy.
(10 Marks)

| $\mathrm{m}_{1}$ | $\mathrm{~m}_{2}$ | $\mathrm{~m}_{3}$ | $\mathrm{~m}_{4}$ | $\mathrm{~m}_{5}$ |
| :---: | :---: | :---: | :---: | :---: |
| $1 / 8$ | $1 / 16$ | $3 / 16$ | $1 / 4$ | $3 / 8$ |

## OR

4 a. A source having alphabet $\mathrm{s}=\left\{\mathrm{s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}, \mathrm{~s}_{4}, \mathrm{~s}_{5}\right\}$ produces a symbols with respective probabilities $1 / 2,1 / 6,1 / 6,1 / 9,1 / 18$.
(i) When the symbols are coded as shown $0,10,110,1110,1111$ respectively.
(ii) When the code is as $00,01,10,110,111$

Find code efficiency and redundancy
(12 Marks)
b. State and prove Kraft McMillan inequality.
(08 Marks)

## Module-3

5 a. Discuss the binary Erasure Channel (BEC) and also derive channel capacity equation for BEC.
(08 Marks)
b. A channel has the following characteristics

$$
\mathrm{P}\left[\frac{\mathrm{Y}}{\mathrm{X}}\right] \begin{array}{cccc}
\mathrm{Y}_{1} & \mathrm{Y}_{2} & \mathrm{Y}_{3} & \mathrm{Y}_{4} \\
\mathrm{X}_{1}\left[\begin{array}{cccc}
\frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\
\mathrm{X}_{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3}
\end{array} \frac{\frac{1}{3}}{}\right]
\end{array}
$$

Find $H(X), H(Y), H(X, Y)$ and channel capacity if $r=1000$ symbols $/ \mathrm{sec}$.
(12 Marks)

## OR

6 a. Determine the rate of transmission of information through a channel whose noise characteristics is as shown in Fig.Q6(a).
Given $P\left(X_{1}\right)=P\left(X_{2}\right)=\frac{1}{2} . \quad$ Assume $r_{s}=10,000$ symbols $/ \mathrm{sec}$.

b. What is mutual information? Mention its properties and prove that $I(X: Y)=H(X)-H\left(\frac{X}{Y}\right) ; \quad I(X: Y)=H(Y)-H\left(\frac{Y}{X}\right)$.
(10 Marks)

## Module-4

7 a. For a $(6,3)$ linear block code the check bits are related to the message bits as per the equations given below:
$\mathrm{c}_{1}=\mathrm{d}_{1} \oplus \mathrm{~d}_{2}$
$\mathrm{c}_{2}=\mathrm{d}_{1} \oplus \mathrm{~d}_{2} \oplus \mathrm{~d}_{3}$
$\mathrm{c}_{3}=\mathrm{d}_{2} \oplus \mathrm{~d}_{3}$
i) Find the generator matrix G
ii) Find all possible code words
iii) Find error detecting and error correcting capabilities of the code.
(12 Marks)
b. The generator polynomial of a $(7,4)$ cyclic code is $g(x)=1+x+x^{2}$. Find the 16 code words of this code by forming the code polynomial $v(x)$ using $V(X)=D(X) G(X)$ where $\mathrm{D}(\mathrm{X})$ is the message polynomial.
(08 Marks)

## OR

8 a. Design a linear block code with a minimum distance of 3 and a message block size of 8 bits.
(08 Marks)
b. For a $(6,3)$ cyclic code, find the following:
(i) $\mathrm{G}(\mathrm{x})$
(ii) G in systematic form
(iii) All possible code words
(iv) Show that every code polynomial is multiple of $\mathrm{g}(\mathrm{x})$.
(12 Marks)

## Module-5

9 a. For the convolution encoder shown in Fig.Q9(a) the information sequence is $\mathrm{d}=10011$. Find the output sequence using the following two approaches.
(i) Time domain approach
(ii) Transfer domain approach


Fig.Q9(a)
(10 Marks)
b. Consider a $(3,1,2)$ convolution encoder with $\mathrm{g}^{(1)}=110, \mathrm{~g}^{(2)}=101$ and $\mathrm{g}^{(3)}=111$.
(i) Draw the encoder diagram
(ii) Find the code word for message sequence (11101) using Generator matrix and Transfer domain approach.
(10 Marks)

## OR

10 a. Consider the rate $\mathrm{r}=\frac{1}{2}$ and constraint length $\mathrm{K}=2$ convolution encoder shown in Fig.Q10(a).
(i) Draw the state diagram.
(ii) Draw the code tree
(iii) Draw Trellis diagram,
(iv) Trace the path through the tree that corresponds to the message sequence $\{1,0,1\}$.


Fig.Q10(a)
(14 Marks)
b. Explain Viterbi decoding.

